Path of a Falling Object

A teenager throws a ball off a rooftop. Assume that the x coordinate of the ball is given by $x(t) = t$ meters and its y coordinate satisfies the following properties:

$$\begin{aligned} y''(t) &= -9.8 \text{ meters/second} \\ y'(0) &= 0 \\ y(0) &= 5 \text{ meters.} \end{aligned}$$

a) Find an equation directly describing y in terms of t.

b) Find a parametrization $(x(t), y(t))$ which describes the path of the ball.

c) Find the speed $\frac{ds}{dt}$ of the ball (this answer will only be valid for times before the ball hits the ground.)

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a) $y''(t) = -9.8$

$$\int y'(t)\,dt = \int -9.8\,dt$$
$$= -9.8t + C$$

$y'(0) = 0 \Rightarrow -9.8(0) + C = 0$
$\Rightarrow C = 0$
$\Rightarrow y'(t) = -9.8t$

$$\int y'(t)\,dt = \int -9.8t\,dt$$
$$= -\frac{9.8t^2}{2} + C$$
$$= -4.9t^2 + C$$

$y(0) = 5 \Rightarrow -4.9(0) + C = 5$
$\Rightarrow C = 5$
$\Rightarrow y(t) = -4.9t^2 + 5$

b) $(t, -4.9t^2 + 5)$

c) $ds = \sqrt{dx^2 + dy^2}$ $\quad \frac{dx}{dt} = 1$

$$\Rightarrow \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \quad \frac{dy}{dt} = y'(t) = -9.8t$$
$$= \sqrt{1 + 96.04t^2}$$